PREVIOUS YEAR SOLVED QUESTION BANK

for

BITS WILP MTECH AIML

MATHEMATICS FOR MACHINE LEARNING

(UNTIL MID SEMESTER - EC2)

Rohith Raj Reganti | crackbitswilp.in

MFML/MFDS Previous Year Solved Question Papers

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Contents

CHAPTER 1

Solution of Linear Systems

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Q 1.1 *8 Marks*

Answer the following for the given matrix:

$$\mathbf{A} = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

(A) Using elementary row operations, write the matrix in its row echelon form

(B) Let **V** be a vector subspace spanned by the columns of matrix **A**. Find the basis and dimension of **V**.

(C) Let **V** be a vector subspace spanned by vectors **x**, such that $\mathbf{Ax} = 0$. Find the basis and dimension of **V**.

(D) Give the set of linearly independent rows of **A**. What is the number of vectors in this set?

Explanation

(A)

$$\mathbf{A} = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2+3*R_1, R_3 \leftarrow R_3-2*R_1, R_2 \leftarrow R_2/5, R_3 \leftarrow R_3-R_2, R_1 \leftarrow R_1-2*R_2}$$

$$\begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NOTE: REF is not unique.

(B) column number 1 and column number 3 have pivot.

Basis of column space is:

$$\begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

Rank of col space = dimension of $\mathbf{V} = 2$

(C) From REF, the null space is:

$$\begin{bmatrix} x \\ r \\ y \\ s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} t$$

basis =

$$\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Rank of null space = dimension of $\mathbf{V} = 3$

(D) From REF, row 1 and row 2 form basis of row space

Rank of row space = 2

Q 1.2 *3 Marks*

Find the value of a such that the rank of A is 3, where

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 0 \\ 0 & 0 & a+9 & 3 \end{bmatrix}$$

Explanation

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 0 \\ 0 & 0 & a+9 & 3 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ a & 2 & 2 & 0 \\ 0 & 0 & a+9 & 3 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ a-2 & 0 & 4 & 0 \\ 0 & 0 & a+9 & 3 \end{bmatrix}$$

$R_4 \to R_4 - 3R_2$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ a-2 & 0 & 4 & 0 \\ 0 & 0 & a+6 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ a-2 & 0 & 0 & -4 \\ 0 & 0 & a+6 & 0 \end{bmatrix}$$

Since $\text{rank}(A) = 3$, one row must become zero.
Therefore,

$$a + 6 = 0$$

$$\boxed{a = -6}$$

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Q 1.3 *5 Marks*

Determine the values of μ for which the linear system $3x - y + \mu z = 0; 2x + y + z = 2; x - 2y - \mu z = -1$ will fail to have unique solution. For this value of μ, are the equations consistent?

Explanation

$$\left.\begin{array}{l} \det = 0 \\ \left.\begin{vmatrix} 3-1 & \mu & \\ 2 & 1 & 1 \\ 1 & -2 & -\mu \end{vmatrix} = 0 \right\} \mid M \\ 3(-\mu-(-2))-(-1)(-2\mu-1)+\mu(-4-1)=0 \\ \Rightarrow 3(-\mu+2)+1(-2\mu-1)+\mu(-5)=0 \\ \Rightarrow -3\mu+6-2\mu-1-5\mu=0 \\ \Rightarrow -10\mu+5=0 \\ \Rightarrow \quad 10\mu=5 \\ \Rightarrow \quad \mu=\frac{5}{10} \\ \Rightarrow \quad \mu=\frac{1}{2} \end{array}\right\} 2M$$

checking for consistency when $\mu = \frac{1}{2}$

$$3x - y + \frac{1}{2}z = 0$$
$$2x + y + z = 2$$
$$x - 2y - \frac{1}{2}z = -1$$

Augmented matrix

$$[A : B] = \begin{bmatrix} 3 & -1 & 1/2 & 0 \\ 2 & 1 & 1 & 2 \\ 1 & -2 & -1/2 & -1 \end{bmatrix}$$

$R_2 \to 3R_2 - 2R_1$ and $R_3 \to 3R_3 - R_1$

$$\begin{bmatrix} 3 & -1 & 1/2 & 0 \\ 0 & 5 & 2 & 6 \\ 0 & -5 & -2 & -3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 3 & -1 & 1/2 & 0 \\ 0 & 5 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$\therefore P[A : B] = 3 \neq P(A) = 2$

$2M$

$\therefore$ Inconsistant

Q 1.4 *3 Marks*

Using the Gauss–Jordan elimination method, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$

Show all the elementary row operations involved in converting the augmented matrix $[A|I]$ into $[I|A^{-1}]$ and hence obtain the inverse of the given matrix.

Explanation

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1 : R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & 0 & 3 & -3 & 1 \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 0 & 3 & -3 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 / -1; R_3 \rightarrow R_3 / -1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 3 & 0 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$R_1 \rightarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Q 1.5 *3 Marks*

Discuss the consistency of the following system of equations and hence find the solution. $x2 + x3 - 2x4 = 0, -4x1 - x2 - x3 + 2x4 = -4, -2x1 + 3x2 + 3x3 - 6x4 = -2$

Explanation

$$011 - 20 - 4 - 1 - 12 - 4 - 233 - 6 - 2$$

After row transformation

$$-4 - 1 - 12 - 4011 - 2000000$$

$$R(A) = R(AB) = 2$$

Infinite Solution is

$$x_1 = 1x_2 = -x_3 + 2x_4x_3 = x_3x_4 = x_4$$

One particular solution is 1,0,0,0

Q 1.6 *8 Marks*

Answer the following for the given matrix:

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(C) Let V be a vector subspace spanned by vectors x , such that Ax = 0. Find the basis and dimension of **V**.

(D) Give the set of linearly independent rows of **A**. What is the number of vectors in this set?

Explanation

(A)

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$$\begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NOTE: REF is not unique.

(B) column number 1 and column number 3 have pivot.
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basis =

$$\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Rank of null space = dimension of $\mathbf{V} = 3$

(D) From REF, row 1 and row 2 form basis of row space Rank of row space $= 2$