MATRIX OPERATIONS

SCALAR ① MULTIPLICATION

ROW ② EXCHANGE

FACTORING OUT COMMON ③ MULTIPLES

① SCALAR MULTIPLICATION

When a matrix is multiplied by a scalar 'k', each element of the matrix is multiplied by k.

e.g., $k = 2$

Let $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \\ 5 & 10 & 15 \end{bmatrix}$ $kA = 2 \times A = \begin{bmatrix} 2 \times 2 & 4 \times 2 & 6 \times 2 \\ 3 \times 2 & 6 \times 2 & 9 \times 2 \\ 5 \times 2 & 10 \times 2 & 15 \times 2 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 6 & 12 & 18 \\ 10 & 20 & 30 \end{bmatrix}$

properties:

• The determinant is scaled by k^n, where n is the size of the matrix

$$\boxed{\det(kA) = k^n \det(A)}$$

for 3×3 matrix $\Rightarrow$ $\det(3A) = 3^3 \det(A)$

• Scalar multiplication doesn't affect the RANK of the matrix.

② ROW EXCHANGE

When two rows are exchanged, the positions of the rows are swapped. eg, we swap rows 1 & 2 of matrix A :

Before Swapping

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \\ 5 & 10 & 15 \end{bmatrix}$$

After Swapping ↓

$$B = \begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \\ 5 & 10 & 15 \end{bmatrix}$$

- The determinant changes sign

  If $\det(A) = d$,

  then $\det(B) = -d$ } Important ✗

- The rank and null space are UNAFFECTED

③ FACTORING OUT COMMON MULTIPLES

When rows of a matrix share a common multiple, we can factor it out. eg.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow[\text{from row } 2]{\text{factor out 2}} \begin{bmatrix} 1 & 3 & 5 \\ 2 \cdot (1 \ 2 \ 3) \\ 7 & 8 & 9 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

- The determinant is scaled by factor 'k' from the row.

- The rank & null space are UNAFFECTED